II. HOOKE AND JEEVES (HOOKE ALGORITHM)*

A. Purpose

This program finds the minimum of a multivariable, unconstrained, nonlinear function:

Minimize \( F(X_1, X_2, \ldots, X_N) \)

B. Method

The procedure is based on the direct search method proposed by Hooke and Jeeves (30). No derivatives are required. The procedure assumes a unimodal function; therefore, if more than one minimum exists or the shape of the surface is unknown, several sets of starting values are recommended. The algorithm proceeds as follows:

1) A base point is picked and the objective function evaluated.
2) Local searches are made in each direction by stepping \( X_i \) a distance \( S_i \) to each side and evaluating the objective function to see if a lower function value is obtained.
3) If there is no function decrease, the step size is reduced and searches are made from the previous best point.
4) If the value of the objective function has decreased, a "temporary head", \( X_{1,i}^{(k+1)} \), is located using the two previous base points \( X_i^{(k+1)} \) and \( X_i^{(k)} \):

\[
X_{1,i}^{(k+1)} = X_i^{(k+1)} + \alpha (X_i^{(k+1)} - X_i^{(k)})
\]

where \( i \) is the variable index = 1, 2, 3, ..., N  
\( \alpha \) denotes the temporary head  
\( k \) is stage index (a stage is the end of N searches)  
\( \alpha \geq 1 \).
5) If the temporary head results in a lower function value, a new local search is performed about the temporary head, a new head is located and the value of \( F \) checked. This expansion continues as long as \( F \) decreases.

*Computer code developed by A. I. Johnson, University of Western Ontario, Canada. Used by permission.
6) If the temporary head does not result in a lower function value, a search is made from the previous best point.

7) The procedure terminates when the convergence criterion is satisfied (see Description of Parameters).

A flow sheet illustrating the above procedure is given in Figure 9.11.

C. Program Description

1) Usage:

The program consists of a short main program, the main subroutine HOOKE and the user supplied functional evaluation subroutine OBJECT. Initial values of the independent variables, step sizes, and solution parameters are supplied through the main program. Subroutine HOOKE performs all searches and provides all printout.

2) Subroutine Required:

SUBROUTINE HOOKE (RK, EPS, NSTAGE, MAXK, NKAT, EPSY, ALPHA, BETA, QD, Q, QQ, W, IPRINT) called from main program, performs all searches.

SUBROUTINE OBJECT (SUMN, AKE, NSTAGE) function evaluation subroutine (user supplied).

3) Description of Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSTAGE</td>
<td>Number of decision variables to be used</td>
</tr>
<tr>
<td>RK</td>
<td>Vector of initial guesses for decision variables</td>
</tr>
<tr>
<td>EPS</td>
<td>Vector of initial step size to be used for each of the variables</td>
</tr>
<tr>
<td>ITMAX</td>
<td>Maximum number of times the objective function is called (=MAXK)</td>
</tr>
<tr>
<td>NKAT</td>
<td>Maximum number of times the initial step size is to be reduced</td>
</tr>
<tr>
<td>EPSY</td>
<td>Error in objective function to be reached before program terminates (difference between current value and previous stage value)</td>
</tr>
<tr>
<td>ALPHA</td>
<td>Factor for extending the size of the initial steps, greater than or equal to 1.0</td>
</tr>
<tr>
<td>BETA</td>
<td>Factor for reducing the initial step size, 0.0 ≤ BETA ≤ 1.0</td>
</tr>
<tr>
<td>QD</td>
<td>Optimum value of the function resulting from the search</td>
</tr>
<tr>
<td>AKE</td>
<td>Vector of independent variables in subroutine OBJECT</td>
</tr>
<tr>
<td>SUMN</td>
<td>Objective function to be minimized - define in OBJECT</td>
</tr>
<tr>
<td>IPRINT</td>
<td>Print control. IPRINT = 0 results in no intermediate output. IPRINT = 1 results in output on each iteration</td>
</tr>
</tbody>
</table>
Figure 9.11. Hooke and Jeeves (HOOKE ALGORITHM) Logic Diagram
NI  Card reader unit number
NO  Printer unit number

4) DIMENSION Requirements:
The DIMENSION statement in main program should be modified according
to the requirements of the particular problem. The parameters
included in the following DIMENSION statement conform to the
Input Parameter definitions above:

DIMENSION EPS (NSTAGE), RK(NSTAGE), Q(NSTAGE), QQ(NSTAGE), W(NSTAGE)

5) Input Formats:

<table>
<thead>
<tr>
<th>CARD TYPE</th>
<th>FORMAT</th>
<th>CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(8I10)</td>
<td>NSTAGE, IPRINT, ITMAX, NKAT</td>
</tr>
<tr>
<td>2</td>
<td>(8E10.4)</td>
<td>(RK(II), II = 1,NSTAGE)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(If NSTAGE &gt; 8, additional CARD TYPE 2's will be required.)</td>
</tr>
<tr>
<td>3</td>
<td>(8E10.4)</td>
<td>(EPS(JJ), JJ = 1,NSTAGE)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(If NSTAGE &gt; 8, additional CARD TYPE 3's will be required.)</td>
</tr>
<tr>
<td>4</td>
<td>(8E10.4)</td>
<td>ALPHA, BETA, EPSY</td>
</tr>
</tbody>
</table>

6) Output:
All output is from subroutine HOOKE. Initial parameter values are
printed. Intermediate results are printed, if the user specifies
IPRINT = 1 on Card Type 1. Final results are printed upon
termination.

7) Summary of User Requirements:
a) Determine values for NSTAGE, IPRINT, ITMAX, NKAT, NI, and NO.
b) Determine initial estimates of independent variables; enter
   as (RK(II), II = 1, NSTAGE).
c) Specify initial step sizes; enter as (EPS(JJ), JJ = 1,NSTAGE).
d) Determine values for ALPHA, BETA, AND EPSY observing the
   rules stated in Description of Parameters section.
e) Adjust DIMENSION in main program.
f) Specify objective function by writing SUBROUTINE OBJECT

g) Adjust FORMAT statements as necessary.

D. Test Problem
The following test program was taken from the literature (13).
Calculations were performed on a CDC 6400 computer.
Function:  \[ F = -3803.84 - 138.08x_1 - 232.92x_2 + 123.08x_1^2 \\
+ 203.64x_2^2 + 182.25x_1x_2 \]

Starting Point:  \( x_1 = 1.0, x_2 = 0.5 \)

Parameters:  \( \text{NSTAGE} = 2, \quad \text{ITMAX} = 500, \quad \text{NKAT} = 20, \)
\( \text{EPSY} = 0.00001, \quad \text{ALPHA} = 1.0, \quad \text{BETA} = 0.5 \)

Initial Step Sizes:  \( \text{EPS}(1) = 0.10, \quad \text{EPS}(2) = 0.10 \)

Algorithm Answers:  \( F = -3873.9 \)
\( x_1 = 0.20576 \)
\( x_2 = 0.47979 \)

Number of Function Evaluations:  110

Central Processor Time:  3 seconds

The listing and output for this problem are contained in the following section.
E. Program Listings and Example Output

**MAIN LINE PROGRAM FOR SUBROUTINE Hooke.**

```
DIMENSION EPS(2), RK(2), Q(2), QQ(2), W(2)
COMMON NI, NO

NI = 50
NO = 66

READ (NI, 001) NSTAGE, IPRINT, ITMAX, NKAT
001 FORMAT (8I10)
READ (NI, 002) (RK(I), I=1,NSTAGE)
002 FORMAT (8E10.4)
READ (NI, 002) (EPS(JJ), JJ=1,NSTAGE)

READ (NI, 003) ALPHA, BETA, EPSY
003 FORMAT (8E10.4)
QD = 0.0

CALL Hooke (RK, EPS, NSTAGE, ITMAX, NKAT, EPSY, ALPHA, BETA, QD, Q, QQ, W, 1 IPRINT)

END
```

**SUBROUTINE Hooke (RK, EPS, NSTAGE, MAXK, NKAT, EPSY, ALPHA, BETA, QD, 1 Q, QQ, W, IPRINT)**

```
DIMENSION RK(NSTAGE), EPS(NSTAGE), Q(NSTAGE), QQ(NSTAGE), 1 W(NSTAGE)
```
COMMON NI,NO

WRITE (NO,001)
001 FORMAT (1H1,10X,37HDOOKE AND JEEVES OPTIMIZATION ROUTINE)
      WRITE (NO,002) ALPHA, BETA, MAXK, NKAT
002 FORMAT (/12X,10HPARAMETERS,/,2X,8HALPHA = ,F5.2,4X,
               17HBETA = ,F5.2,4X,8HITMAX = ,I4,4X,7HNKAT = ,I3)
      WRITE (NO,003) NSTAGE
003 FORMAT (/12X,22HNUMBER OF VARIABLES = ,I3)
      WRITE (NO,004) N
004 FORMAT (/12X,18HINITIAL STEP SIZES)
      DO 6 I=1,NSTAGE
      WRITE (NO,005) I, EPS(I)
005 FORMAT (/12X,4HEPS(,,I2,4H) = ,E16.8)
      6 CONTINUE
      WRITE (NO,007) EPSY
007 FORMAT (/12X,43HError IN FUNCTION VALUES FOR CONVERGENCE = ,E16.8)
      KFLAG = 0
      DO 601 I=1,NSTAGE
      Q(I) =RK(I)
      W(I) = 0.0
      601 CONTINUE
      KAT = 0.0
      KK1 = 0
      KCOUNT = 0
      WBEST = W(NSTAGE)
      CALL OBJECT (SUM,RK,NSTAGE)
      KK1 = KK1+ 1
      BO =SUM
      IF (KK1.EQ. 1) QD = SUM
      IF (KK1.EQ. 1) GO TO 201
      IF (BO.GT.QD) KFLAG = 1
      IF (BO.LT.QD) QD = BO

C C
C ESTABLISHING THE SEARCH PATTERN
C
201 DO 55 I = 1,NSTAGE
      QQ(I)=RK(I)
      TSRK = RK(I)
      RK(I) = RK(I) + EPS(I)
      CALL OBJECT (SUM,RK,NSTAGE)
      KK1 = KK1+ 1
      W(I) = SUM
      IF (W(I) .LT.QD) GO TO 58
      RK(I) = RK(I) - 2.0*EPS(I)
      CALL OBJECT (SUM,RK,NSTAGE)
      KK1 = KK1+ 1
      W(I) = SUM
      IF (W(I) .LT.QD) GO TO 58
      RK(I) = TSRK
      IF (I.EQ. 1) GO TO 513
      W(I) =W(I-1)
GO TO 613
513 W(I) = BO
613 CONTINUE
   KCOUNT = 1 + KCOUNT
   GO TO 55
58 QD = W(I)
59 QQ(I) = RK(I)
55 CONTINUE
   IF (IPRINT) 60, 65, 60
60 WRITE (NO,100) KK1
   RECORD RESPONSES AND LOCATION
   WRITE(NO,102)
   WRITE(NO,207) (RK(I), I=1,NSTAGE), QD
   TEST TO DETERMINE TERMINATION OF PROGRAM
65 IF (KK1.GT.MAXK) GO TO 94
66 IF (KAT .GE. NKAT) GO TO 94
67 IF (ABS(W(NSTAGE)-WBEST).LE.EPSY) GO TO 94
68 IF ALL AXES FAIL REDUCE STEP SIZE
   IF (KCOUNT .GE. NSTAGE) GO TO 28
   DO 26 I = 1,NSTAGE
   RK(I) = RK(I) + ALPHA*(RK(I) - Q(I))
   26 CONTINUE
   DO 25 I = 1,NSTAGE
   Q(I) = QQ(I)
   25 CONTINUE
   GO TO 70
   REDUCE STEP SIZE
   KAT = KAT + 1
   IF (KFLAG .EQ. 1) GO TO 202
   GO TO 204
202 KFLAG = 0
   DO 203 I = 1,NSTAGE
   RK(I) = Q(I)
   203 CONTINUE
   DO 80 I=1,NSTAGE
   EPS(I) = EPS(I) * BETA
   80 CONTINUE
   IF (IPRINT) 85, 70, 85
85 WRITE (NO,101) KAT
   GO TO 70
94 WRITE (NO,460) (EPS(I), I=1,NSTAGE)
   WRITE (NO,461) (RK(I), I=1,NSTAGE)
   WRITE (NO,462) QD
   DO 104 I=1,NSTAGE
104 WRITE (NO,103) I, RK(I)
WRITE (NO,100) KK1
100 FORMAT (2X,12HNUMBER OF FUNCTION EVALUATIONS = ,I8)
101 FORMAT (2X,18HSTEP SIZE REDUCED ,12,6H TIMES)
102 FORMAT (1X,26HEND OF EACH PATTERN SEARCH/)
103 FORMAT (2X,18HFINDAL X(12,4H) = ,1PE16.8)
207 FORMAT (1X,18HVARIABLES AND SUMN,3X,9E12.4/)
465 FORMAT (10X,3HSUM,3X,E14.5)
466 FORMAT (1X,18H THE FINAL EPS ARE, 4F20.8/)
467 FORMAT (1X,18H THE FINAL RK ARE , 5F20.8/)
468 FORMAT (1X,24H THE MINIMUM RESPONSE IS, F20.8/)
RETURN
END

SUBROUTINE OBJECT (SUMN,AKE,NSTAGE)

DIMENSION AKE(NSTAGE)

X1 = AKE(1)
X2 = AKE(2)
X12 = (X1**2)
X22 = (X2**2)
SUMN = 3803.84 * 138.08*X1 + 232.92*X2 - 123.08*X1*X2 - 203.64
1 *X2**2 - 182.25*X1*X2
SUMN = - SUMN
RETURN
END
HOOKE AND JEEVES OPTIMIZATION ROUTINE

PARAMETERS
ALPHA = 1.00  BETA = 0.50  ITMAX = 500  NKAT = 20

NUMBER OF VARIABLES = 2

INITIAL STEP SIZES
EPS( 1) = 0.10000000E+00
EPS( 2) = 0.10000000E+00

ERROR IN FUNCTION VALUES FOR CONVERGENCE = 0.10000000E-04

NUMBER OF FUNCTION EVALUATIONS = 5
END OF EACH PATTERN SEARCH

VARIABLES AND SUMN  0.9000E+00  0.4000E+00  -3.823E+04

(9 intervening printouts are omitted.)

NUMBER OF FUNCTION EVALUATIONS = 50
END OF EACH PATTERN SEARCH

VARIABLES AND SUMN  0.2250E+00  0.4750E+00  -3.874E+04

STEP SIZE REDUCED 4 TIMES
(11 intervening printouts are omitted.)

NUMBER OF FUNCTION EVALUATIONS = 106
END OF EACH PATTERN SEARCH

VARIABLES AND SUMN 0.2059E+00 0.4797E+00 -.3874E+04

STEP SIZE REduced 10 TIMES

NUMBER OF FUNCTION EVALUATIONS = 110
END OF EACH PATTERN SEARCH

VARIABLES AND SUMN 0.2058E+00 0.4798E+00 -.3874E+04
THE FINAL EPS ARE 0.00009766 0.00009766
THE FINAL RK ARE 0.20576172 0.47978316
THE MINIMUM RESPONSE IS -3873.92354660

FINAL X(1) = 2.0576172E-01

FINAL X(2) = 4.7978516E-01

NUMBER OF FUNCTION EVALUATIONS = 110